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COLLOQUIUM

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Cartan type estimates for certain vector-valued potentials

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4:15 p.m., ES 143

Tea at 3:45 p.m., ES 152

ABSTRACT

For given complex numbers $\nu_1, \ldots, \nu_N$, points $z_1, \ldots, z_N$ in the complex plane and $P > 0$ we define the set

$$\mathcal{Z}(P, \nu) = \left\{ z \in \mathbb{C} : \left| \sum_{j=1}^{N} \frac{\nu_j}{z - z_j} \right| > P \right\}$$

(for $\nu_j \equiv 1$ the sum is the logarithmic derivative of the polynomial $\prod_{j=1}^{N}(z - z_j)$ and has also the certain physical interpretation). Our goal is to estimate the size of $\mathcal{Z}(P, \nu)$ in terms of the radii of covering disks. The sharp estimate (up to an absolute constant) of the set

$$\mathcal{X}(P, \nu) = \left\{ z \in \mathbb{C} : \sum_{j=1}^{N} \left| \frac{\nu_j}{z - z_j} \right| > P \right\}$$

can be easily obtained using a method of Cartan (1928). Thus, the problem is to catch the effect of the mutual cancellation of the terms when we pass from the sum of moduli to the modulus of the sum.

In spite of the fact that the problem on estimation of $\mathcal{Z}(P, \nu)$ was posed by Macintyre and Fuchs in 1940, it was solved only in 2005 by J. M. Anderson and the speaker using a tool which appeared only in the last 12 years in connection with the development of the theory of analytic capacity (Melnikov, Tolsa, Mattila, Nazarov, Treil, Volberg and others).

In the lecture I will tell about some of the notions and facts of this theory as well as about a very recent generalization of the problem described above and its applications.