HOMEWORK ASSIGNMENT 1
AMAT326 (SPRING 09)

Due: Feb 3 (Tuesday)

(1) Verify that the relation, two ordered pairs (a, b) and (c, d) are equivalent if $ad = bc$, is an equivalence relation on the set S, where $S = \{(a, b) : a, b \in \mathbb{Z}, b \neq 0\}$.

(2) Prove that $1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$ for all $n \geq 1$.

(3) (i) Prove that for any $n \geq 1$,

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^{n-1} + \frac{x^n}{1-x}.$$

(ii) Substituting $x = 2$ in (i), prove that for any $n \geq 1$,

$$1 + 2 + 2^2 + \cdots + 2^{n-1} = 2^n - 1.$$

(4) What is wrong with the following theorem and proof?

Theorem *All babies have the same color eyes.*

Proof. $n = 1$ is obvious. Suppose that in any set of n babies all have the same color eyes. Consider a set of $n+1$ babies. We may ssume by induction that in the set L of n babies to the left all have the same color eyes, and similarly that in the set R of the n babies to the right all have the same color eyes. But then evidently all the $n + 1$ babies have the same color eyes, for the leftmost and the rightmost babies have the same color eyes as all the babies in between. By indction, for any n, in every set of n babies all have the same color eyes. Since the set of all babies is one such set, the theorem is proved.

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