HOMEWORK ASSIGNMENT 3
AMAT326 (S09)

Due: Feb 24 (Tuesday)

(1) Show that if n is not prime, n has a prime divisor $\leq \sqrt{n}$.

(2) If $(a, b) = p^3$, p a prime, what is (a^2, b^2)?

(3) Show that if $(a, m) = d$ and $(b, m) = 1$, then $(ab, m) = d$.

(4) Show that $[a, b] = ab$ iff a and b are relatively prime.

(5) Find the least common multiple of $3,630,000 = 2^4 \cdot 5^3 \cdot 11^2 \cdot 15$ and $915,062,500 = 4 \cdot 55^4 \cdot 25$.

(6) Use the idea of Euclid's proof to prove that there are infinitely many primes of the form $4n - 1$ (*Hint:* Consider $4p_1 \cdots p_r - 1$).

(7) Show that $F(m + 1) = F(m)(F(m) - 2) + 2$.

(8) Prove that for any n there exist n consecutive natural numbers none of which are prime (*Hint:* Start with $(n + 1)! + 2$).

(9) Find the least nonnegative residue of $5^{18} \pmod 7$.

(10) Show that for nay two integers a, b, $(a + b)^2 \equiv a^2 + b^2 \pmod 2$.

Department of Mathematics and Statistics, University at Albany, Albany, NY 12222