

HOMEWORK ASSIGNMENT 5
AMAT326 (S09)

Due: April 14 (Tuesday)

- (1) Find the orders of the units of $\mathbb{Z}/9\mathbb{Z}$
- (2) Find the order of $[32]$ in $\mathbb{Z}/17\mathbb{Z}$ (*Hint*: Use Proposition 3 on page 137, 2nd edition)
- (3) Show that the order of $10 \pmod{83}$ is at least 30.
- (4) Suppose that F is a field, and a is a nonzero element of F . Show that if $ab = ac$ for some $b, c \in F$, then $b = c$.
- (5) Consider the set of numbers $a + bi$ where $a, b \in \mathbb{Z}/2\mathbb{Z}$. Here $i^2 = -1$. Write down all four elements of this set. Which elements have inverses?
- (6) Show that if $f : R \rightarrow S$ is a ring homomorphism and if a is a unit of R , then $f(a)$ is a unit of S . Show, in fact, that $f(a^{-1}) = f(a)^{-1}$ for any unit a of R .
- (7) Show that if R is a ring, the function f from R to $M_2(R)$, the set of 2×2 matrices with entries in R , given by $f(r) = \begin{pmatrix} r & 0 \\ 0 & r \end{pmatrix}$ for any $r \in R$, is a homomorphism.
- (8) Find the exponent g_0 of G , where $G = U_7$, the group of units in $\mathbb{Z}/7\mathbb{Z}$.

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