PRACTICE PROBLEMS
AMAT326 (S09)

(1) Show that if p is prime, then $\binom{p-1}{k} \equiv (-1)^k \pmod{p}$ for $1 \leq k \leq p-1$.
Hint : Use Wilson's theorem.

(2) Find the least positive integer x such that

$$\begin{aligned} x &\equiv 5 \pmod{7}, \\ x &\equiv 7 \pmod{11}, \\ x &\equiv 3 \pmod{13}. \end{aligned}$$

(3) Find a primitive root modulo 23.

(4) Let p be an odd prime, how many solutions are there to $x^{p-1} \equiv 1 \pmod{p}$; to $x^{p-1} \equiv 2 \pmod{p}$?

(5) What is the smallest positive integer m such that the multiplicative group modulo m is not cyclic?

(6) Find the value of $\left(\frac{3}{17}\right)$.

(7) Prove that 5 is a quadratic residue modulo 11. List all solutions of $x^2 \equiv 5 \pmod{11}$.

(8) Decide whether $x^2 \equiv 150 \pmod{1009}$ is solvable or not.

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