

ABSTRACT. A finite von Neumann algebra  $\mathcal{M}$  with a faithful normal trace  $\tau$  has Haagerup's approximation property (relative to a von Neumann subalgebra  $\mathcal{N}$ ) if there exists a net  $(\varphi_\alpha)_{\alpha \in \Lambda}$  of normal completely positive ( $\mathcal{N}$ -bimodular) maps from  $\mathcal{M}$  to  $\mathcal{M}$  that satisfy the subtracial condition  $\tau \circ \varphi_\alpha \leq \tau$ , the extension operators  $T_{\varphi_\alpha}$  are bounded compact operators (in  $\langle \mathcal{M}, e_{\mathcal{N}} \rangle$ ), and pointwise approximate the identity in the trace-norm, i.e.,  $\lim_\alpha \|\varphi_\alpha(x) - x\|_2 = 0$  for all  $x \in \mathcal{M}$ . We prove that the subtraciality condition can be removed, and provide a description of Haagerup's approximation property in terms of Connes's theory of correspondences. We show that if  $\mathcal{N} \subseteq \mathcal{M}$  is an amenable inclusion of finite von Neumann algebras and  $\mathcal{N}$  has Haagerup's approximation property, then  $\mathcal{M}$  also has Haagerup's approximation property. This work answers two questions of Sorin Popa.