

Algebraic Stein Varieties

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It is well-known that the associated analytic space of an affine variety defined over complex number field is Stein but the converse is not true, that is, an algebraic Stein variety is not necessarily affine. In this talk, we will give sufficient and necessary conditions for an algebraic Stein variety to be affine. One of our results is that a quasi-projective variety Y defined over complex number field with dimension d ($d \geq 1$) is affine if and only if Y is Stein, $H^i(Y, \mathcal{O}_Y) = 0$ for all $i > 0$ and $\kappa(D, X) = d$ (i.e., Y has d algebraically independent nonconstant regular functions), where X is a projective variety containing Y , D is an effective divisor with support $X - Y$, \mathcal{O}_Y is the sheaf of regular functions on Y and the cohomology is Čech cohomology. If Y is Stein but not affine, we will also discuss the possible transcendental degree of the nonconstant regular functions on Y . We will show that Y cannot have $d - 1$ algebraically independent nonconstant regular functions. The interesting phenomenon is that the transcendental degree can be even if the dimension of Y is even and the degree can be odd if the dimension of Y is odd.